Exam. Code : 103204
Subject Code : 1121

B.A./B.Sc. 4th Semester

MATHEMATICS

Paper—II

(Solid Geometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Answer any *five* questions, selecting at least *two* questions from each section.

SECTION—A

1. (a) Show that the equation of the cone whose vertex is the origin and whose base is the circle through the three points (a, 0, 0), (0, b, 0), (0, 0, c) is $\Sigma a(b^2 + c^2)yz = 0$. 5

(b) Find the equation of the right circular cone generated by straight line drawn from origin to cut the circle through the points (1, 2, 2), (2, 1, –2), (2, –2, 1). 5

2. (a) Find the equation of the elliptic cone whose vertex is origin and which intersects the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=c.$$ 5

(b) Prove that the equation $x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$ represents a cone, find its vertex. 5

3. (a) If $x = \frac{1}{2}, y = z$ represents one of a set of three mutually perpendicular generators of the cone $11yz + 6zx - 14xy = 0$, find the equation of other two. 5

(b) Find the angle between the lines given by

$$x + y + z = 0 \text{ and } \frac{yz}{q-r} + \frac{zx}{r-p} + \frac{xy}{p-q} = 0.$$ 5

4. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$. 5

(b) Find the equation of the right circular cylinder whose guiding circle is

$x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0,$

$2x + 3y + 6z = 0.$ 5

5. (a) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y = 1$ having its generators parallel to the line $x = y = z$. 5

(b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 5

SECTION—B

6. (a) Find the equation of the surface generated by revolution of the circle $x^2 + y^2 - 2ay + a^2 - r^2 = 0$, $z = 0$ about the x-axis $(a > r)$. 5

(b) Identify the surface

$$4x^2 + 9y^2 + z^2 - 6x + 6y - 4z + 10 = 0.$$ 5

7. Reduce the equation

$$3x^2 + 7y^2 + 3z^2 + 10yz - 2zx + 10xy + 4x - 12y - 4z + 1 = 0$$

to the standard form and state the nature of the surface represented by it. 10

8. (a) A tangent plane to ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the co-ordinates axes in L, M, N. Prove that the centroid of the triangle LMN lies on

$$\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9.$$ 5

(b) The normal at any point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meet the principal planes in G_1, G_2, G_3. Show that $PG_1 : PG_2 : PG_3 = a^2 : b^2 : c^2$. 5

9. (a) Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$. 5

(b) Show that if the origin is the centre of a conicoid, the coefficients of the first degree terms in its equation are all zero. 5

10. Prove that the normal from (α, β, γ) to paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone

$$\frac{\alpha}{x-\alpha}-\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0.$$ 10